Reg. No. :

Question Paper Code : D 2288

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2010.

Second Semester

Mechanical Engineering

MA 1151 — MATHEMATICS — II

(Regulation 2004)

(Common to all branches of B.E./B.Tech. Except Food Technology)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\int_1^b \int_1^a \frac{dxdy}{xy}$.

2. Change the order of integration in $\int_0^a \int_0^y f(x,y)\,dy\,dx$.

3. Define 'grad ϕ' and give its geometrical meaning.

4. Define divergence and curl of a vector point function.

5. Verify if $e^x \sin y$ is Harmonic.

6. What is the image of $0 \le x \le 2$ under the transformation $w = iz$.

7. State Cauchy's integral formula.

8. Find the Taylor's series of $\frac{z-1}{z^2}$ about $z = 1$.

9. Define Laplace Transforms and the conditions for its existance.

10. Find $L^{-1}\left(\frac{1}{(s+2)^4}\right)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the area between the circle $x^2 + y^2 = a^2$ and the line $x + y = a$ by double integration. (8)

(ii) Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z} dx\,dy\,dz$. (8)

Or

(b) (i) By transforming into cylindrical coordinates, evaluate $\iiint (x^2 + y^2 + z^2)\, dx\,dy\,dz$ taken over $x^2 + y^2 \le 1$ and $0 \le z \le 1$. (8)

(ii) Evaluate : $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y} dy\,dx$. (8)

12. (a) (i) Find the directional derivative of $\Phi = x^2yz + 4xz^2$ at $p(1,-2,-1)$ that is maximum and also in the direction of PQ where Q is $(3,-3,-2)$. (8)

(ii) Verify divergence theorem for $\overline{F} = x^2i + y^2j + z^2k$ where S is the surface formed by the planes $x = 0$, $x = a$, $y = 0$, $y = b$, $z = 0$ and $z = c$. (8)

Or

(b) (i) Show that $\overline{F} = (y^2 + 2xz^2)i + (2xy - z)j + (2x^2z - y + 2z)k$ is irrotational and find its scalar potential. (8)

(ii) Evaluate $\int_C \phi d\overline{V}$ where C is the curve $x = t$, $y = t^2$, $z = (1-t)$ and $\phi = x^2y(1+z)$ from $t = 0$ to $t = 1$. (8)

13. (a) (i) Show that an analytic function with constant real part is constant and an analytic function with constant modulus is also constant. (8)

(ii) Find the bilinear transformation which maps $z = i,-1,1$ into $w = 0,1,\infty$ respectively. (8)

Or

(b) (i) Find the analytic function $w = u + iv$, if $u = e^x(x \sin y + y \cos y)$. Hence find V. (8)

(ii) Show that the transformation $w = \cos z$, maps the segment of the X-axis $0 \le x \le \pi/2$ into $0 \le u \le 1$ of the u-axis. (8)

14. (a) (i) Evaluate $\int_C \frac{z\,dz}{(z-1)(z-2)^2}$ where 'C' is the contour $|z-2| < \frac{1}{2}$ using Integral formula. (8)

(ii) Find the Laurent expansion of $\frac{1}{z(1-z)}$ valid in the region $|z+1|<1$ and $|z+1|>2$. (8)

Or

(b) (i) Using Residue theorem, evaluate $\int_C \frac{z+1}{z(z-1)}dz$ where C is $|z|=2$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}\,(a>b>0)$ using contour integration. (8)

15. (a) (i) Find the Laplace Transform of $te^{-t}\cos^t$ and $\frac{e^{-t}-e^{-2t}}{t}$. (8)

(ii) Find the Laplace Transform of $f(t)=\begin{cases} K\ 0\le t\le a \\ -K, a\le t\le 2a. \end{cases}$ and $f(t)=f(t+2a)$ for all 't'. (8)

Or

(b) (i) Find the inverse Laplace transform of $\frac{2s-9}{s^2+6s+34}$ and $\cot^{-1}\left(\frac{2}{s+1}\right)$. (8)

(ii) Using Laplace Transform, solve

$$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=6x^2e^{-3x}.$$ (8)

Given that when $x = 0$, $y = 0$ and $\frac{dy}{dx}=0$.