Reg. No. :

Question Paper Code : 11484

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2012.

Second Semester

Civil Engineering

MA 2161/MA 22/080030004 — MATHEMATICS — II

(Common to all branches)

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the Wronskian of y_1, y_2 of $y'' - 2y' + y = e^x \log x$.

2. Find the particular integral of $(D^2 - 4D + 4)y = 2^x$.

3. Prove that $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is irrotational.

4. State Gauss divergence theorem.

5. Show that the function $f(z) = \bar{z}$ is nowhere differentiable.

6. Find the map of the circle $|z| = 3$ under the transformation $w = 2z$.

7. Evaluate $\int_C \frac{z\,dz}{(z-1)(z-2)}$, where C is the circle $|z| = 1/2$.

8. If $f(z) = \frac{-1}{z-1} - 2[1 + (z-1) + (z-1)^2 + \cdots]$, find the residue of $f(z)$ at $z = 1$.

9. Is the linearity property applicable to $L\left\{\frac{1-\cos t}{t}\right\}$? Reason out.

10. Find the inverse Laplace transform of $\frac{1}{(s+1)(s+2)}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve the equation $(D^2 + 5D + 4)y = e^{-x}\sin 2x$. (8)

(ii) Solve the equation $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ by the method of variation of parameters. (8)

Or

(b) (i) Solve $\frac{dx}{dt} + y = e^t$, $x - \frac{dy}{dt} = t$. (8)

(ii) Solve the equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log x}{x^2}$. (8)

12. (a) (i) Show that $\overline{F} = (2xy - z^2)\overline{i} + (x^2 + 2yz)\overline{j} + (y^2 - 2zx)\overline{k}$ is irrotational and find its scalar potential. (8)

(ii) Verify Green's theorem for $\overline{V} = (x^2 + y^2)\overline{i} - 2xy\overline{j}$ taken around the rectangle bounded by the lines $x = \pm a$, $y = 0$ and $y = b$. (8)

Or

(b) Verify Gauss's divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$. (16)

13. (a) (i) Find the bilinear transformation that maps the points $z = \infty, i, 0$ onto $w = 0, i, \infty$ respectively. (8)

(ii) Determine the analytic function whose real part is $\dfrac{\sin 2x}{\cosh 2y - \cos 2x}$. (8)

Or

(b) (i) Find the image of the hyperbola $x^2 - y^2 = 1$ under the transformation $w = \dfrac{1}{z}$. (8)

(ii) Prove that the transformation $w = \dfrac{z}{1-z}$ maps the upper half of z-plane on to the upper half of w-plane. What is the image of $|z| = 1$ under this transformation? (8)

14. (a) (i) Evaluate $\int_C \dfrac{z+4}{z^2 + 2z + 5}\, dz$, where C is the circle $|z + 1 + i| = 2$, using Cauchy's integral formula. (8)

(ii) Find the residues of $f(x) = \dfrac{z^2}{(z-1)^2(z+2)^2}$ at its isolated singularities using Laurent's series expansions. Also state the valid region. (8)

Or

(b) Evaluate $\displaystyle\int_0^{2\pi} \frac{\sin^2\theta}{a + b\cos\theta}\, d\theta$, $a > b > 0$. (16)

15. (a) (i) Find $L^{-1}\left[\dfrac{s^2}{(s^2+4)^2}\right]$ using convolution theorem. (8)

(ii) Find the Laplace transform of the Half wave rectifier

$$f(t) = \begin{cases} \sin wt, & 0 < t < \pi/w \\ 0, & \pi/w < t < 2\pi/w \end{cases}$$ and $f(t + 2\pi/w) = f(t)$ for all t. (8)

Or

(b) (i) Find $L\left[\dfrac{\cos at - \cos bt}{t}\right]$. (8)

(ii) Solve $\dfrac{d^2x}{dt^2} - 3\dfrac{dx}{dt} + 2x = 2$, given $x = 0$ and $\dfrac{dx}{dt} = 5$ for $t = 0$ using Laplace transform method. (8)
