Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 10394

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2012.

Second Semester

Common to all branches

MA 2161/181202/MA 22/080030004 — MATHEMATICS – II

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Transform the equation $(2x+3)^2 \frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 6x$ into a differential equation with constant coefficients.
2. Find the particular integral of $(D-1)^2 y = e^x \sin x$.
3. Find λ such that $\vec{F} = (3x-2y+z)\vec{i} + (4x+\lambda y - z)\vec{j} + (x-y+2z)\vec{k}$ is solenoidal.
4. State Gauss divergence theorem.
5. State the basic difference between the limit of a function of a real variable and that of a complex variable.
6. Prove that a bilinear transformation has atmost two fixed points.
7. Define singular point.
8. Find the residue of the function $f(z) = \frac{4}{z^3(z-2)}$ at a simple pole.
9. State the first shifting theorem on Laplace transforms.
10. Verify initial value theorem for $f(t) = 1 + e^{-t}(\sin t + \cos t)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve $(D^2 + a^2)y = \sec ax$ using the method of variation of parameters. (8)

(ii) Solve : $(D^2 - 4D + 3)y = e^x \cos 2x$. (8)

Or

(b) (i) Solve the differential equation $(x^2D^2 - xD + 4)y = x^2 \sin(\log x)$. (8)

(ii) Solve the simultaneous differential equations $\frac{dx}{dt} + 2y = \sin 2t$, $\frac{dy}{dt} - 2x = \cos 2t$. (8)

12. (a) (i) Show that $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$ is irrotational and hence find its scalar potential. (8)

(ii) Verify Green's theorem in a plane for $\int_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$, where C is the boundary of the region defined by $x = 0$, $y = 0$ and $x + y = 1$. (8)

Or

(b) (i) Using Stoke's theorem, evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = y^2\vec{i} + x^2\vec{j} - (x+z)\vec{k}$ and 'C' is the boundary of the triangle with vertices at (0, 0, 0), (1, 0, 0), (1, 1, 0). (8)

(ii) Find the work done in moving a particle in the force field given by $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$ along the straight line from (0,0,0) to (2, 1, 3). (8)

13. (a) (i) Prove that every analytic function $w = u + iv$ can be expressed as a function of z alone, not as a function of $\bar{z}$. (8)

(ii) Find the bilinear transformation which maps the points $z = 0, 1, \infty$ into $w = i, 1, -i$ respectively. (8)

Or

(b) (i) If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f(z)| = 0$. (8)

(ii) Show that the image of the hyperbola $x^2 - y^2 = 1$ under the transformation $w = \frac{1}{z}$ is the lemniscate $r^2 = \cos 2\theta$. (8)

14. (a) (i) Evaluate $\int_C \frac{z\,dz}{(z-1)(z-2)^2}$ where C is $|z-2| = \frac{1}{2}$ by using Cauchy's integral formula. (8)

(ii) Evaluate $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series valid for the regions $|z| > 3$ and $1 < |z| < 3$. (8)

Or

(b) (i) Evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)}\,dz$, where C is the circle $|z-i| = 2$ using Cauchy's residue theorem. (8)

(ii) Evaluate $\int_0^\infty \frac{\cos mx}{x^2 + a^2}\,dx$, using contour integration. (8)

15. (a) (i) Apply convolution theorem to evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$. (8)

(ii) Find the Laplace transform of the following triangular wave function given by $f(t) = \begin{cases} t, & 0 \le t \le \pi \\ 2\pi - t, & \pi \le t \le 2\pi \end{cases}$ and $f(t+2\pi) = f(t)$. (8)

Or

(b) (i) Find the Laplace transform of $\frac{e^{at} - e^{-bt}}{t}$. (4)

(ii) Evaluate $\int_0^\infty te^{-2t}\cos t\,dt$ using Laplace transform. (4)

(iii) Solve the differential equation $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{-t}$ with $y(0) = 1$ and $y'(0) = 0$, using Laplace transform. (8)
