Reg. No. :

Question Paper Code : 51569

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2014.

Second Semester

Civil Engineering

MA 2161/MA 22/080030004 — MATHEMATICS — II

(Common to all branches)

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Solve $(D^4 - 2D^2 + 1)y = 0$.
2. Guess the trial solution of the particular integral for the differential equation $y'' + 4y = \cos 2x$ using method of undetermined coefficients.
3. Find the directional derivative of $\phi = x^2 + y^2 + z^2$ at the point (1, 1, 1) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.
4. If $\overline{F} = \overline{\nabla}\phi$, then find $\int_A^B \overline{F} \cdot d\overline{r}$.
5. Verify whether or not $f(z) = e^x(\cos y - i \sin y)$ is analytic.
6. Find the image of $|z - 10i| = 2$ under the mapping $w = z + 1 + i$.
7. Evaluate $\int_C \frac{5z^2 + 30z + 100}{(z-2)} dz$, where C is the circle $|z - 2| = 4$.
8. Identify and classify the singularity of the function $f(z) = e^{1/z}$.
9. Find the Laplace transform of $f(t) = t \cos ht$.
10. Find the inverse Laplace transform of $\frac{e^{-\pi s}}{(s-1)^2}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Using method of variation of parameters solve the following differential equation $y'' - 4y' + 4y = (1 + x)e^{2x}$. (8)

(ii) Solve $(x^2D^2 - 3xD + 4)y = x[\log x]^2$. (8)

Or

(b) (i) Solve $(x+1)^2 y'' + (x+1)y' + y = 2\sin[\log(1+x)]$. (8)

(ii) Solve the following differential equation by method of undetermined coefficients $y'' + y = 4e^x + 10\sin x$. (8)

12. (a) (i) Verify Green's theorem for $\int_C \left[(xy + y^2)dx + x^2 dy\right]$ where C is the boundary of the common area between $y = x^2$ and $y = x$. (8)

(ii) Verify Stoke's theorem for the vector field $\overline{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where S is the surface of upper hemisphere $x^2 + y^2 + z^2 = 1$ and C is its boundary in xy-plane. (8)

Or

(b) (i) Verify Gauss divergence theorem for the vector function $\overline{F}=x^3\hat{i}+y^3\hat{j}+z^3\hat{k}$, over the cubic region bounded by $x=0$, $x=a$, $y=0$, $y=a$, $z=0$ and $z=a$. (10)

(ii) Verify that $\overline{F}=y^2\hat{i}+2xy\hat{j}+2z\hat{k}$ is irrotational, further find also its corresponding scalar potential. (6)

13. (a) (i) Find the analytic function $f(z)=u(x,y)+iv(x,y)$ given that $u-v=e^x(\cos y-\sin y)$. (8)

(ii) Find the image of the region bounded by the lines $x=0$, $y=0$ and $x+y=1$ under the mappings $w=e^{i\pi/4}z$ and $w=z+(2+3i)$. (8)

Or

(b) (i) Find the image of the circle $|z-3i|=3$ and the region $1<x<2$ under the map $w=1/z$. (8)

(ii) Find the bilinear transformation which maps the points $z=1, i, -1$ into the points $w=0,1,\infty$ respectively. Find also the pre-image of $|w|=1$ under this bilinear transformation. (8)

14. (a) (i) If $f(a)=\int_C \frac{13z^2+27z+15}{z-a}dz$ where C is $|z|=2$, then find $f(3)$, $f'(1-i)$, $f''(1-i)$ and $f(1-i)$. (8)

(ii) Using Contour integration on unit circle, evaluate $\int_0^{2\pi}\frac{d\theta}{(5+4\cos\theta)}$. (8)

Or

(b) (i) Using Contour integration, evaluate $\int_{-\infty}^{\infty}\frac{x^2}{(x^2+1)(x^2+9)}dx$. (8)

(ii) Find the Laurent's series expansion of $f(z)=\frac{7z-2}{(z-2)(z+1)}$ valid in the regions $|z+1|<1$ and $|z+1|>3$. (8)

15. (a) (i) Solve $y''-6y'+9y=t^2e^{3t}$, $y(0)=2$, $y'(0)=6$ by Laplace transform method. (8)

(ii) Using convolution theorem find the inverse Laplace transform of $\frac{s}{(s^2+1)^2}$. (8)

Or

(b) (i) Verify initial and final value theorems for the function $f(t)=1+e^{-t}(\sin t+\cos t)$. (8)

(ii) Find the Laplace transform of the periodic function defined on the interval $0\le t\le 1$ by $f(t)=\begin{cases}-1, & 0\le t<1/2\\ 1, & 1/2\le t<1\end{cases}$ and $f(t+1)=f(t)$. (8)

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