Reg. No. :

Question Paper Code : 97104

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

Second Semester

Civil Engineering

MA 6251 — MATHEMATICS — II

(Common to all branches except Marine Engineering)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find $curl\,\vec{F}$ if $\vec{F} = xy\hat{i} + yz\hat{j} + zx\hat{k}$.

2. State Stoke's theorem.

3. Find the particular integral of $(D^2 + 2D + 2)y = e^{-x}\sin 2x$.

4. Solve: $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

5. Find the Laplace transform of $e^{-t}\sin 2t$.

6. Find $f(t)$ if the Laplace transform of $f(t)$ is $\frac{s}{(s+1)^2}$.

7. Verify $f(z) = z^3$ is analytic or not.

8. Find the critical points of the transformation $w^2 = (z-\alpha)(z-\beta)$.

9. Evaluate $\int_C \frac{z\,dz}{z-2}$ where C is the circle $|z| = 1$.

10. Find the residue of $f(z) = \frac{z^2}{(z-2)(z+1)^2}$ at $z = 2$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the angle between the normals to the surface $x^2 = yz$ at the points $(1, 1, 1)$ and $(2, 4, 1)$. (8)

(ii) Using Green's theorem, evaluate $\int_C \left[\left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy\right]$ where C is the boundary of the triangle formed by the lines $x = 0$. $y = 0$, $x + y = 1$ in the xy - plane. (8)

Or

(b) (i) Verify divergence theorem for $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ taken over the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. (10)

(ii) Prove that $\vec{F} = \left(x^2 - y^2 + x\right)\hat{i} - (2xy + y)\hat{j}$ is irrotational and hence find its scalar potential. (6)

12. (a) (i) Solve: $\left(D^2 + 2D + 5\right)y = e^{-x}.x^2$. (8)

(ii) Solve: $\left(x^2D^2 - xD + 1\right)y = \sin\left(\log x\right)$. (8)

Or

(b) (i) Using method of variation of parameters, solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$. (8)

(ii) Solve: $\frac{dx}{dt} + y = e^t$; $x - \frac{dy}{dt} = t$. (8)

13. (a) (i) Find the Laplace transform of the following functions

(1) $e^{-t}\, t \cos t$

(2) $\frac{1 - \cos t}{t}$. (8)

(ii) Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s^2}{\left(s^2 + a^2\right)\left(s^2 + b^2\right)}\right\}$. (8)

Or

(b) (i) Find the Laplace transform of $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$, $f(t + 2a) = f(t)$. (8)

(ii) Use Laplace transform to solve $\left(D^2 - 3D + 2\right)y = e^{3t}$ with $y(0) = 1$ and $y'(0) = 0$. (8)

14. (a) (i) If $f(z)$ is an analytic function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2 .$$ (8)

(ii) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. (8)

Or

(b) (i) Prove that $u = x^2 - y^2$ and $v = -\frac{y}{x^2+y^2}$ are harmonic functions but not harmonic conjugates. (8)

(ii) Given that $u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$, find the analytic function $f(z) = u + iv$. (8)

15. (a) (i) Evaluate $\int_C \frac{z+1}{z^2+2z+4}\,dz$ where C is the circle $|z+1+i| = 2$ using Cauchy's integral formula. (8)

(ii) Find the Laurent's series expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ valid in the regions $|z| > 2$ and $0 < |z-1| < 1$. (8)

Or

(b) (i) Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\cos\theta}$ using contour integration. (8)

(ii) Evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2+9)(x^2+4)}$ using contour integration. (8)

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