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Question Paper Code : P 1421

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

Seventh Semester

Mechanical Engineering

ME 1401 — INTRODUCTION OF FINITE ELEMENT ANALYSIS

(Regulation 2004)

(Common to Automobile Engineering and Mechatronics Engineering)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Distinguish between 1D bar element and 1D beam element.

2. What is Galerkin method of approximation?

3. What are CST & LST elements?

4. What is a shape function?

5. State the properties of stiffness matrix.

6. Write down the governing differential equation for a two dimensional steady-state heat transfer problem.

7. What is meant by axi-symetric field problem? Give an example.

8. Distinguish between plane stress and plane strain problems.

9. What are the differences between 2 Dimensional scalar variable and vector variable elements?

10. Distinguish between essential boundary conditions and natural boundary conditions.

PART B — (5 × 16 = 80 marks)

11. (a) (i) What is constitutive relationship? Express the constitutive relations for a linear elastic isotropic material including initial stress and strain. (6)

(ii) Consider the differential equation $(d^2y/dx^2)+400x^2=0$ for $0 \le x \le 1$ subject to boundary conditions $y(0)=0; y(1)=0$.

The functional corresponding to this problem, to be extremized is given by

$$I=\int_0^1 \{-0.5(dy/dx)^2+400x^2y\}$$

Find the solution of the problem using Rayleigh-Ritz method by considering a two-term solution as $y(x) = c_1x(1-x) + c_2x^2(1-x)$. (10)

Or

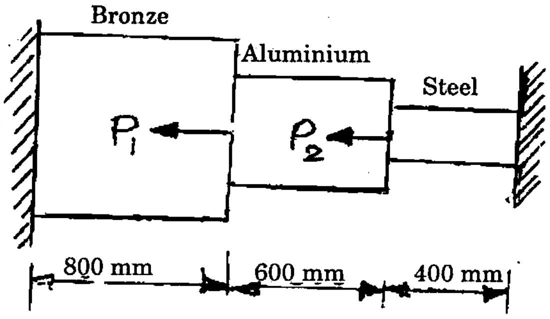
(b) (i) A physical phenomenon is governed by the differential equation $(d^2w/dx^2)-10x^2=5$ for $0 \le x \le 1$.

The boundary conditions are given by $w(0)=w(1)=0$. By taking a two-term trial solution as $w(x)=C_1f_1(x)+C_2f_2(x)$ with $f_1(x)=x(x-1)$ and $f_2(x)=x^2(x-1)$, find the solution of the problem using the Galerkin method. (10)

(ii) Solve the following system of equations using Gauss elimination method. (6)

$$x_1+3x_2+2x_3=13$$
$$-2x_1+x_2-x_3=-3$$
$$-5x_1+x_2+3x_3=6$$

12. (a) The stepped bar shown in Fig. 1 is subjected to an increase in temperature, $\Delta T = 80°C$. Determine the displacements, element stresses and support reactions.



$\alpha_B = 18.9\times10^{-6}/°C$

$\alpha_{Al} = 23\times10^{-6}/°C$

$\alpha_S = 11.7\times10^{-6}/°C$

$P_1 = 60 kN$

$P_2 = 75 kN$

$\Delta T = 80°C$

Fig. 1.

	Bronze	Aluminimum	Steel
$A =$	2400 mm²	1200 mm²	600 mm²
$E =$	83 GPa	70 GPa	200 GPa

Or

(b) Consider a two-bar truss supported by a spring shown in Fig. 2. Both bars have E = 210 GPa and $A = 5.0 \times 10^{-4}\, m^2$. Bar one has a length of 5 m and bar two has a length of 10 m. The spring stiffness is k = 2 kN/m. Determine the horizontal and vertical displacements at the joint 1 and stresses in each bar.

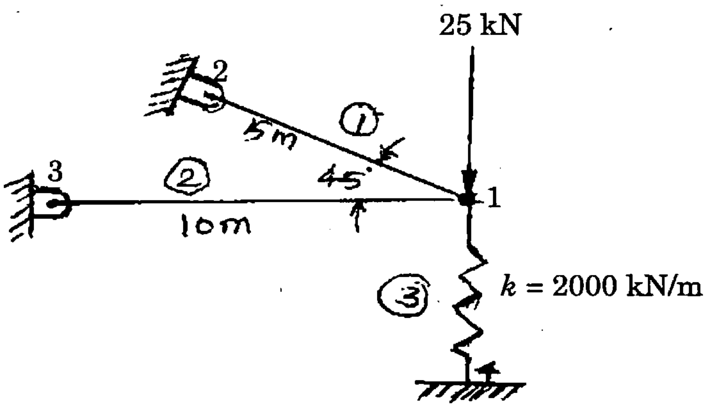


Fig. 2.

13. (a) (i) The (x, y) co-ordinates of nodes, i, j, and k of a triangular element are given by (0, 0), (3, 0) and (1.5, 4) mm respectively. Evaluate the shape functions N_1, N_2 and N_3 at an interior point P (2, 2.5) mm for the element. (4)

(ii) For the same triangular element, obtain the strain-displacement relation matrix B. (12)

Or

(b) Compute element matrices and vectors for the element shown in Fig. 3, when the edge kj experiences convection heat loss.

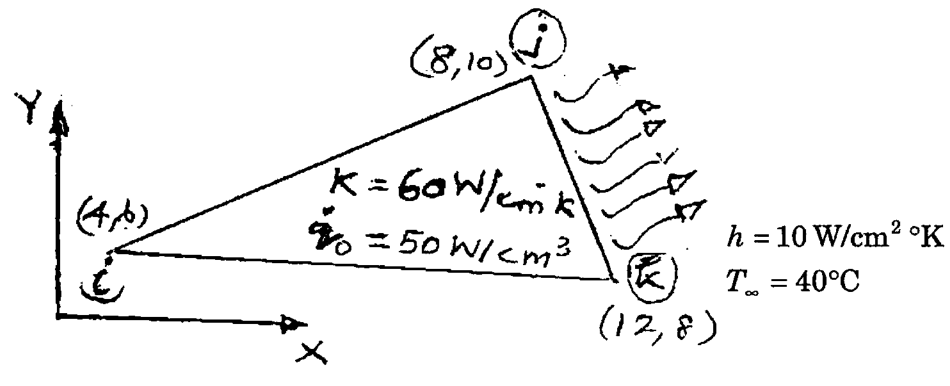
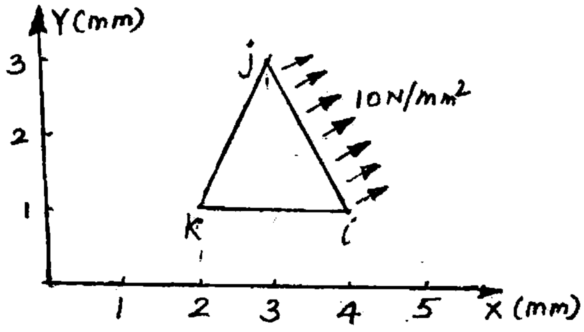


Fig. 3.

14. (a) The triangular element shown in Fig. 4 is subjected to a constant pressure 10 N/mm^2 along the edge ij. Assume E = 200 GPa, Poisson's ratio μ = 0.3 and thickness of the element = 2 mm. The coefficient of thermal expansion of the material is $\alpha = 2\times10^{-6}/°C$ and $\Delta T = 50°C$. Determine the constitutive matrix (stress-strain relationship matrix D) and the nodal force vector for the element.



Thickness = 2 mm
E = 200 GPa
μ = 0.3
$\alpha = 2\times10^{-6}/°C$
ΔT = 50°C

Fig. 4.

Or

(b) The (x, y) co-ordinates of nodes, i, j, and k of an axisymmetric triangular element are given by (3, 4), (6, 5) and (5, 8) cm respectively. The element displacement (in cm) vector is given as q = [0.002, 0.001, 0.001, 0.004, –0.003, 0.007]^T. Determine the element strains.

15. (a) (i) The Cartesian (global) coordinates of the corner nodes of a quadrilateral element are given by (0, –1), (–2, 3), (2, 4) and (5, 3). Find the coordinate transformation between the global and local (natural) coordinates. Using this, determine the Cartesian coordinates of the point defined by (r, s) = (0.5, 0.5) in the global coordinate system. (8)

(ii) Evaluate the integral

$$I = \int_{-1}^{1} (2 + x + x^2)\,dx$$ and compare with exact results. (8)

Or

(b) (i) The Cartesian (global) coordinates of the corner nodes of an isoparametric quadrilateral element are given by (1, 0), (2, 0), (2.5, 1.5) and (1.5, 1). Find its Jacobian matrix. (12)

(ii) Distinguish between subparametric and superparametric elements. (4)

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