Question Paper Code : 21521

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013.

Second Semester

Civil Engineering

MA 2161/MA 22/080030004 — MATHEMATICS — II

(Common to all branches)

(Regulation 2008)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the particular integral of $(D^2 - 2D + 1)y = \cosh x$.

2. Solve $x^2 \dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} + 2y = 0$.

3. Find the directional derivative of $\phi = xyz$ at $(1,1,1)$ in the direction of $\vec{i} + \vec{j} + \vec{k}$.

4. If $\vec{A}$ and $\vec{B}$ are irrotational, prove that $\vec{A} \times \vec{B}$ is solenoidal.

5. Find the image of the line $x = k$ under the transformation $w = \dfrac{1}{z}$.

6. Find the fixed points of mapping $w = \dfrac{6z - 9}{z}$.

7. Evaluate $\displaystyle\int_C \frac{3z^2 + 7z + 1}{z + 1}\, dz$, where C is $|z| = \dfrac{1}{2}$.

8. Find the residue of $\dfrac{1 - e^{2z}}{z^4}$ at $z = 0$.

9. Find the Laplace transform of $\dfrac{t}{e^t}$.

10. Verify initial value theorem for the function $f(t) = ae^{-bt}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve the differential equation $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = \dfrac{e^{-x}}{x^2}$ by the method of variation of parameters. (8)

(ii) Solve : $(3x + 2)^2 \dfrac{d^2y}{dx^2} + 3(3x + 2)\dfrac{dy}{dx} - 36y = 3x^2 + 4x + 1$. (8)

Or

(b) (i) Solve the simultaneous differential equations : $\dfrac{dx}{dt} + 5x - 2y = t$; $\dfrac{dy}{dt} + 2x + y = 0$. (8)

(ii) Solve $x^2 \dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} + 2y = x^2 + \dfrac{1}{x^2}$. (8)

12. (a) Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ over the upper half surface $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy – plane. (16)

Or

(b) Verify divergence theorem for $\vec{F} = x^2\vec{i} + z\vec{j} + yz\vec{k}$ over the cube formed by the planes $x = \pm 1$, $y = \pm 1, z = \pm 1$. (16)

13. (a) (i) Prove that the function $u = e^x(x\cos y - y\sin y)$ satisfies Laplace's equation and find the corresponding analytic function $f(z) = u + iv$. (8)

(ii) Find the Bilinear transformation which maps $z = 0$, $z = 1$, $z = \infty$ into the points $w = i$, $w = 1, w = -i$. (8)

Or

(b) (i) Find the image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$. (8)

(ii) If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (8)

14. (a) (i) Expand the function $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ in Laurent's series for $|z| > 3$. (8)

(ii) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where C is $|z| = 3$. (8)

Or

(b) (i) Evaluate $\int_0^\infty \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}, a > 0, b > 0$. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$ using contour integration. (8)

15. (a) (i) Find $L[t^2 e^{-3t} \sin 2t]$. (8)

(ii) Find the Laplace transform of the square-wave function (or Meoander function) of period a defined as (8)

$$f(t) = \begin{cases} 1, & \text{when } 0 < t < \frac{a}{2} \\ -1, & \text{when } \frac{a}{2} < t < a. \end{cases}$$

Or

(b) (i) Using convolution theorem find the inverse Laplace transform of $\frac{4}{(s^2 + 2s + 5)^2}$. (8)

(ii) Solve $y'' + 5y' + 6y = 2$ given $y'(0) = 0$ and $y(0) = 0$ using Laplace transform. (8)