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**Question Paper Code : 41310**

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Third Semester

Civil Engineering

MA 6351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS  
(Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/  
Agriculture Engineering/Automobile Engineering/Biomedical Engineering/  
Computer Science and Engineering/Computer and Communication Engineering/  
Electrical and Electronics Engineering/Electronics and Communication  
Engineering/Electronics and Instrumentation Engineering/Geoinformatics  
Engineering/Industrial Engineering/Industrial Engineering and Management/  
Instrumentation and Control Engineering/Manufacturing Engineering/Marine  
Engineering/Materials Science and Engineering/Mechanical Engineering/  
Mechanical and Automation Engineering/Mechatronics Engineering/Medical  
Electronics/Petrochemical Engineering/Production Engineering/Robotics and  
Automation Engineering/Bio Technology/Chemical Engineering/Chemical  
and Electrochemical Engineering/Food Technology/Information Technology/  
Petrochemical Technology/Petroleum Engineering/Plastic Technology/  
Polymer Technology)  
(Regulations 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A

(10×2=20 Marks)

1. Find the complete integral of the PDE :  $z = px + qy + \sqrt{pq}$
2. Solve :  $(D^3 - 3DD'^2 + 2D'^3) z = 0$
3. Find  $b_n$  in the expansion of  $f(x) = x^2$  as a Fourier series in  $(-\pi, \pi)$ .
4. Define Root mean square value of a function.
5. Classify the partial differential equation  $u_{xy} = u_x u_y + xy$ .
6. State possible solutions of the one dimensional heat equation.
7. Find the Fourier transform of  $f(x) = \begin{cases} 1 & \text{in } |x| < a \\ 0 & \text{in } |x| > a \end{cases}$



8. State the convolution theorem for Fourier transforms.
9. Find the Z-transform of  $\{n\}$ .
10. Prove that  $Z\{nf(n)\} = -z \frac{d}{dz} F(z)$ , where  $Z\{f(n)\} = F(z)$ .

## PART - B

(5×16=80 Marks)

11. a) i) Find the singular solution of the equation  $z = px + qy + p^2 + pq + q^2$ . (8)

ii) Solve :  $x(y - z) p + y(z - x) q = z(x - y)$ . (8)

(OR)

- b) i) Solve :  $(D^2 + 4DD' - 5D'^2)z = \sin(x - 2y) + e^{2x - y}$ . (8)

ii) Solve :  $(D^2 + DD' - 6D'^2)z = y \cos x$ . (8)

12. a) i) Find the Fourier series for  $f(x) = x^2$  in  $-\pi < x < \pi$ . (8)

ii) Find the half range cosine series for  $f(x) = x(\pi - x)$  in  $(0, \pi)$ . (8)

(OR)

- b) Find the Fourier series expansion upto the first three harmonics for the function defined in the following table (16)

|   |   |                 |                  |       |                  |                  |        |
|---|---|-----------------|------------------|-------|------------------|------------------|--------|
| x | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ | $2\pi$ |
| y | 1 | 1.4             | 1.9              | 1.7   | 1.5              | 1.2              | 1.0    |

13. a) A string is stretched and fastened to two points  $x = 0$  and  $x = l$  apart. Motion is started by displacing the string into the form  $y = k(lx - x^2)$  from which it is released at time  $t = 0$ . Find the displacement of any point on the string at a distance of  $x$  from one end at time  $t$ . (16)

(OR)

- b) A bar, 10 cm long with insulated sides, has its ends A and B kept at  $20^\circ\text{C}$  and  $40^\circ\text{C}$  respectively until steady state conditions prevail. The temperature at A is then suddenly raised to  $50^\circ\text{C}$  and at the same instant that at B is lowered to  $10^\circ\text{C}$ . Find the subsequent temperature at any point of the bar at any time. (16)



14. a) Find the Fourier transform of the function  $f(x)$  defined by  $f(x) = \begin{cases} 1-x^2 & \text{in } |x| \leq 1 \\ 0 & \text{in } |x| > 1 \end{cases}$ .

Hence prove that  $\int_0^\infty \frac{\sin s - s \cos s}{s^3} \cos\left(\frac{s}{2}\right) ds = \frac{3\pi}{16}$ . Also show that

$$\int_0^\infty \frac{(x \cos x - \sin x)^2}{x^6} dx = \frac{\pi}{15}. \quad (16)$$

(OR)

- b) i) Show that the Fourier transform of  $f(x) = e^{-\frac{x^2}{2}}$  is  $e^{-\frac{s^2}{2}}$ . (8)

ii) Find the Fourier cosine transform of  $e^{-s^2 x^2}$  and hence find the Fourier sine transform of  $xe^{-s^2 x^2}$ . (8)

15. a) i) Find the inverse Z-transform of  $\frac{8z^2}{(2z-1)(4z+1)}$  using convolution theorem for Z-transforms. (8)

ii) Find the inverse Z-transform of  $\frac{z^2 - 3z}{(z-5)(z+2)}$  using residue theorem. (8)

(OR)

- b) i) Solve :  $y_{n+2} - 4y_{n+1} + 4y_n = 0$ ,  $y_0 = 1$ ,  $y_1 = 0$ , using Z-transform. (10)

ii) Find the Z-transform of  $\{n\}$  and  $\left\{\frac{1}{n+1}\right\}$ . (6)