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Question Paper Code : 90342

REE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Fourth Semester

Computer Science and Engineering

MA8402 – PROBABILITY AND QUEUEING THEORY

(Regulations 2017)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions

PART – A

(10×2=20 Marks)

- ii) Derive the steady state system size probabilities for a $M/M/1/N/FCFS$ queueing model and hence obtain the mean number of customers in the queue. (8)
- (OR)
- b) i) Derive the steady-state system-size probabilities for a $M/M/C/\infty$ FCFS queueing model and hence obtain the mean number of customers in the system. (8)
- ii) Patients arrive at a clinic according to a Poisson process at a rate of 3 patients per hour. The waiting room cannot accommodate more than 6 patients. Examination time per patient is exponentially distributed random variable with rate of 4 per hour.
- 1) Find the effective arrival rate at the clinic.
 - 2) What is the probability that an arriving patient will not wait?
 - 3) What is the expected waiting time W_s in the system? (8)
15. a) Discuss an $M/G/1/\infty$ FCFS queueing system and hence obtain the Pollaczek-Khintchine (P-K) mean value formula for the system size. Deduce also the mean number of customers in the system for $M/M/1/\infty$ FCFS queueing model from the P-K mean value formula. (16)
- (OR)
- b) Derive the system of differential difference equations for the joint probabilities of the system size of two-station tandem queueing system. Under the steady-state conditions, determine the steady-state probabilities of the system size and obtain 1) Expected number of customers in the system, 2) The mean waiting time in the system. (16)

1. A bag contains 8 white and 4 black balls. If 5 balls are drawn at random, what is the probability that 3 are white and 2 are black?
2. Let $M_X(t) = \frac{1}{1-t}$, $|t| < 1$, be the moment generating function of a R.V. X . Find $E(X)$ and $E(X^2)$.
3. If $f(x, y) = e^{-(x+y)}$, $x \geq 0$, $y \geq 0$, is the joint probability density function of (X, Y) , Find $P(X + Y \leq 1)$.
4. Let X and Y be independent R.Vs with $\text{Var}(X) = 9$ and $\text{Var}(Y) = 3$. What is $\text{Var}(4X - 2Y + 6)$?
5. Define : Markov process.
6. Let $\{X_n; n \geq 0\}$ be a Markov chain having state space $S = \{1, 2\}$ and one-step TPM $P = \begin{bmatrix} 1/2 & 1/2 \\ 0 & 1 \end{bmatrix}$. Find the stationary probabilities of the Markov chain.
7. In an $M/M/1/\infty/FCFS$ queue, the service rate, $\mu = \frac{1}{3}$ / minute and waiting time in the queue $W_q = 3$ minute, compute the arrival rate, λ .
8. For a $M/M/C/N/FCFS$ ($C < N$) queueing system, write the expressions for P_0 and P_N .

9. In an M/D/1 queueing system, an arrival rate of customers is 1/6 per minute and the server takes exactly 4 minutes to serve a customer. Calculate the mean number of customers in the system.
10. For an open Jackson queueing network, write the expression for traffic equations and stability condition of the system.

PART - B

(5×16=80 Marks)

11. a) i) There are 3 boxes containing respectively, 1 white, 2 red, 3 black balls, 2 white, 3 red, 1 black balls; 3 white, 1 red, 2 black balls. A box is chosen at random and from it two balls are drawn at random. The two balls are 1 red and 1 white. What is the probability that they came from second box? (8)
- ii) The p.d.f. of a continuous R.V. X is given by $f(x) = \begin{cases} \frac{x}{2} e^{-\frac{x}{2}}, & x > 0 \\ 0, & x \leq 0 \end{cases}$. Obtain:
- C.D.F. of X, F(x)
 - $P(X > 1)$
 - $P(1 < X < 2)$
 - $E(X^2)$.
- (OR)
- b) i) Let X be a binomial R.V with $E(X) = 4$ and $\text{Var}(X) = 3$. Find: (1) $P(X = 5)$, (2) M.G.F. of X, $M_X(t)$, (3) $E(X^2 - 1)$, (4) $\text{Var}\left(-\frac{1}{2}X + 4\right)$. (8)
- ii) A R.V. X is uniformly distributed on $(-5, 15)$. Determine:
- C.D.F. of X, F(x)
 - $P(X < 5/X > 0)$
 - $P(|X - 1| < 5)$
 - $E\left(e^{-\frac{x}{2}}\right)$.
- (8)
12. a) i) The joint p.d.f. of (X, Y) is given by $f(x, y) = \begin{cases} \frac{1}{240}, & 8.5 \leq x \leq 10.5, 120 \leq y \leq 240 \\ 0, & \text{otherwise} \end{cases}$. Obtain:
- The marginal p.d.f.s of X and Y.
 - $E(X)$ and $E(Y)$
 - $E(XY)$
 - Are X and Y independent R.Vs? Justify.
- (8)
- ii) Let X and Y be two continuous R.Vs with joint p.d.f.
- $$f(x, y) = \begin{cases} 4xy, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$
- Determine the joint p.d.f. of the R.Vs $U = X^2$ and $V = XY$ and hence obtain the marginal p.d.f. of U. (8)
- (OR)

- b) i) The joint p.d.f. of R.V (X, Y) is given as $f(x, y) = \begin{cases} Ce^{-(2x+3y)}, & 0 \leq y \leq x < \infty \\ 0, & \text{otherwise} \end{cases}$. Find:
- The value of C.
 - Are the R.Vs X and Y independent? (8)
- ii) Let X and Y be random variables such that $E(X) = 1$, $E(Y) = 2$, $\text{Var}(X) = 6$, $\text{Var}(Y) = 9$ and the correlation coefficient $\rho_{XY} = -\frac{2}{3}$. Calculate:
- The covariance, $\text{Cov}(X, Y)$, of X and Y
 - $E(XY)$
 - $E(X^2)$ and $E(Y^2)$. (8)
13. a) i) Consider a random process $X(t) = \cos(t + \phi)$, where ϕ is a R.V. such that $P(\phi = 0) = P(\phi = \pi) = \frac{1}{2}$. Determine 1) $E(X(t))$, 2) $E(X^2(t))$, 3) $R_{XX}(t, t + \tau)$. Is the process $X(t)$ wide-sense stationary? Justify. (8)
- ii) State the postulates of a Poisson process $\{X(t); t \geq 0\}$ with parameter λ . Derive the system of differential difference equations and hence obtain the probability distribution, $P(X(t) = n) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$, $n = 0, 1, 2, \dots$ (8)
- (OR)
- b) i) Let $\{X_n; n \geq 0\}$ be a Markov chain having state space $S = \{1, 2, 3\}$ with one-step TPM. $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{bmatrix}$.
- Draw a transition diagram.
 - Is the chain irreducible? Explain.
 - Is the state - 2 ergodic? Justify your answer. (8)
- ii) Let $X(t)$ and $Y(t)$ be two independent Poisson processes with parameters λ_1 and λ_2 respectively. Obtain 1) $P(X(t) + Y(t) = n)$, $n = 0, 1, 2, \dots$, 2) $P(X(t) - Y(t) = n)$, $n = 0, \pm 1, \pm 2, \dots$ (8)
14. a) i) A petrol station has one petrol pump. The cars arrive for service according to a Poisson process at a rate of 0.5 cars per minute and the service time for each car follows the exponential distribution with rate of 1 car per minute. compute:
- The probability that the pump station is idle
 - The probability that 10 or more cars are in the system
 - The mean number, L_s , of cars in the system.
 - The mean waiting time, W_q , in the queue and the mean waiting time, W_s , in the system. (8)