SUBJECT NAME   *: Statistics and Numerical Methods*

SUBJECT CODE   *: MA6452*

MATERIAL NAME   *: University Questions*

REGULATION   *: R2013*

UPDATED ON   *: November 2017*   *(Upto N/D 2017 Q.P)*

Unit – I (Testing of Hypothesis)

• Student's 't' test

1. A random sample of 10 boys has the following IQ's 70, 83, 88, 95, 98, 100, 101, 107, 110 and 120. Do these data support the assumption of a population mean IQ of 100 at 5% level of significance? (N/D 2017)

2. The heights of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches? (A/M 2011)

3. A sample of 10 boys had the I.Q's: 70, 120, 110, 101, 88, 83, 95, 98, 100 and 107. Test whether the population mean I.Q may be 100. (N/D 2012)

4. The IQ's of 10 girls are respectively 120, 110, 70, 88, 101, 100, 83, 98, 95, 107. Test whether these two proportions are same. (M/J 2016)

5. The height of six randomly chosen sailors are (in inches): 63, 65, 68, 69, 71 and 72. Those of 10 randomly chosen soldiers are 61, 62, 65, 66, 69, 69, 70, 71, 72 and 73. Discuss, the height that these data thrown on the suggestion that sailors are on the average taller than soldiers ($t_{0.05}(14) = 1.76$). (N/D 2014)

• F – test

1. Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables. (M/J 2014)

Sample I:	18	13	12	15	12	14	16	14	15
Sample II:	16	19	13	16	18	13	15		

   Do the estimates of population variance differ significantly at 5% level of significance?

2. Time taken by workers in performing a job are given below:

Type I	21	17	27	28	24	23	-
Type II	28	34	43	36	33	35	39

   Test whether there is any significant difference between the variances of time distribution. (N/D 2013)

P.KUPPAN M.Sc., B.Ed., M.Phil

3. Test whether there is any significant difference between the variances of the populations from which the following samples are taken: (N/D 2012), (N/D 2017)

     Sample I: 20 16 26 27 23 22
     Sample II: 27 33 42 35 32 34 38

4. Test if the difference in the means is significant for the following data: (N/D 2010)

Sample I:	76	68	70	43	94	68	33	
Sample II:	40	48	92	85	70	76	68	22

5. Two random samples gave the following results:

Sample	Size	Sample mean	Sum of squares of deviation from the mean
1	10	15	90
2	12	14	108

Test whether the samples have come from the same normal population. (M/J 2012)

• **Chi-Square test (Goodness of fit)**

1. A dice is thrown 400 times and a throw of 3 or 4 is observed 150 times. Test the

hypothesis that the dice is fair. (M/J 2012)

2. Theory predicts that the proportion of beans in four groups A, B, C, D should be 9:3:3:1. In an experiment among 1600 beans, the numbers in the four groups were 882, 313, 287 and 118. Does the experiment support the theory? (M/J 2012),(M/J 2016)

3. 4 coins were tossed 160 times and the following results were obtained:
     No. of heads: 0   1   2   3   4
     Observed frequencies: 17 52 54 31 6
Under the assumption that the coins are unbiased, find the expected frequencies of getting 0, 1, 2, 3, 4 heads and test the goodness of fit. (A/M 2011)

4. The following data gives the number of aircraft accidents that occurred during the various days of a week. Find whether the accidents are uniformly distributed over the week. (N/D 2010)

Days:	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents:	14	16	8	12	11	9	14

5. The demand for a particular spare part in a factory was found to vary from day-to-day. In a sample study the following information was obtained.

Days:	Mon	Tues	Wed	Thurs	Fri	Sat
No. of spare parts demanded:	1124	1125	1110	1120	1126	1115

Test the hypothesis that the number of parts demanded does not depend on the day of the week. ($\chi^2_{0.05}(5) = 11.07$) (N/D 2014)

• Chi-Square test (Independence of attributes)

1. Using the data given in the following table to test at 1% level of significance whether a person's ability in Mathematics is independent of his/her interest in Statistics.

(N/D 2017)

		Ability in Mathematics		
		Low	Average	High
Interest in Statistics	Low	63	42	15
	Average	58	61	31
	High	14	47	29

2. Out of 8000 graduates in a town 800 are females, out of 1600 graduate employees 120 are females. Use χ^2 to determine if any distinction is made in appointment on the basic of sex. Value of χ^2 at 5% level for one degree of freedom is 3.84. (A/M 2010)

3. An automobile company gives you the following information about age groups and the liking for particular model of car which it plans to introduce. On the basic of this data can it be concluded that the model appeal is independent of the age group.
($\chi^2_{0.05}(3) = 7.815$) (A/M 2010)

Persons who:	Below 20	20 – 39	40 – 59	60 and above
Liked the car:	140	80	40	20
Disliked the car:	60	50	30	80

4. Test of the fidelity and selectivity of 190 radio receivers produced the results shown in the following table:

	Fidelity		
Selectivity	Low	Average	High
Low	6	12	32
Average	33	61	18

P.KUPPAN M.Sc., B.Ed., M.Phil

High	13	15	0

Use the 0.01 level of significance to test whether there is a relationship between fidelity and selectivity. (A/M 2011)

- ## Large Sample (n > 30)
  ### Single Mean & Difference of Means:

1. A sample of 900 members has a mean 3.4 cm and standard deviation 2.61 cm. Is the sample from a large population of mean 3.25 cms and standard deviation of 2.61cms? (Test at 5% level of significance. The value of z at 5% level is $|z_\alpha| < 1.96$).(A/M 2010)

2. The means of two large samples of 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same populations of standard deviation 2.5 inches? (M/J 2012)

3. A mathematics test was given to 50 girls and 75 boys. The girls made an average grade of 76 with a SD of 6, while boys made an average grade of 82 with a SD of 2. Test whether there is any significant difference between the performance of boys and girls. (N/D 2012),(M/J 2016)

4. A random sample of 100 bulbs from a company P shows a mean life 1300 hours and standard deviation of 82 hours. Another random sample of 100 bulbs from company Q showed a mean life 1248 hours and standard deviation of 93 hours. Are the bulbs of company P superior to bulbs of company Q at 5% level of significance? (N/D 2017)

5. The sales manager of a large company conducted a sample survey in two places A and B taking 200 samples in each case. The results were the following table. Test whether the average sales is the same in the 2 areas at 5% level. (N/D 2013)

	Place A	Place B
Average Sales	Rs. 2,000	Rs. 1,700
S.D	Rs. 200	Rs. 450

6. Examine whether the difference in the variability in yields is significant at 5% level of significance, for the following. (N/D 2010)

	Set of 40 plots	Set of 60 plots
Mean yield per plot	1256	1243
S.D. per plot	34	28

P.KUPPAN M.Sc., B.Ed., M.Phil

Single Proportion & Difference of Proportions:

7. 20 people were attacked by a disease and only 18 survived. Will you reject the hypothesis that the survival rate, if attacked by this disease is 85% is favor of the hypothesis that is more at 5% level? (N/D 2013)

8. A Manufacturer of light bulbs claims that an average of 2% of the bulbs manufactured by him are defective. A random sample of 400 bulbs contained 13 defective bulbs. On the basis of the sample, can you support the manufacturer's claim at 5% level of significance? (M/J 2014)

9. 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test whether these two proportions are same. (M/J 2016)

10. In a random sample of 100 men taken from village A, 60 were found to be consuming alcohol. In another sample of 200 men taken from village B, 100 were found to be consuming alcohol. Do the two villages differ significantly in respect of the proportion of men who consume alcohol? (M/J 2014)

11. Before an increase in excise duty on tea, 800 persons out of a sample of 1000 persons were found to be tea drinkers. After an increase in duty, 800 people were tea drinkers in a sample of 1200 people. Using standard error of proportion, state whether there is a significant decrease in the consumption of tea after the increase in excise duty.
(z_α at 5% level 1.645, 1% level 2.33). (A/M 2010)

12. A machine puts out 16 imperfect articles in a sample of 500. After it was overhauled, it puts out 3 imperfect articles in a sample of 100. Has the machine improved in its performance? (N/D2012)

13. A machine produces 16 imperfect articles in a sample of 500. After machine is overhauled, it produces 3 imperfect articles in a batch of 100. Has the machine been improved? (N/D 2010)

14. Before an increase in excise duty on tea, 900 persons out of a sample of 1100 persons were found to be tea drinkers. After an increase in excise duty, 900 person were tea drinkers in a sample of 1300. Using standard error of proportion, state whether there is a significant decrease in the consumption of tea after the increase in excise duty? (N/D 2013)

15. In a random sample of 1000 people from city A, 400 are found to be consumers of wheat. In a sample of 800 from city B, 400 are found to be consumers of wheat. Does this data give a significant difference between the two cities as far as the proportion of wheat consumers is concerned? (A/M 2011)

P.KUPPAN M.Sc., B.Ed., M.Phil

- ## Theory Questions

1. Explain clearly the procedure generally followed in testing of a hypothesis.   (N/D 2014)

2. Explain briefly the procedure involved in testing the significance for difference of proportions in the case of large samples.   (N/D 2014)

Unit – II (Design of Experiments)

- ## Completely Randomized Design (One – Way Classification)

1. The following are the number of mistakes made in 5 successive days by 4 technicians working for a photographic laboratory test at a level of significance $\alpha = 0.01$. Test whether the difference among the four sample means can be attributed to chance.   (A/M 2011)

Technician			
I	II	III	IV
6	14	10	9
14	9	12	12
10	12	7	8
8	10	15	10
11	14	11	11

- ## Randomized Block Design (Two – Way Classification)

1. Three varieties of coal were analysed by 4 chemists and the ash content is tabulated here. Perform an anatysis of variance.   (M/J 2016)

		Chemists			
		A	B	C	D
Coal	I	8	5	5	7
	II	7	6	4	4
	III	3	6	5	4

2. Analyse the following RBD and find your conclusion. (N/D 2013)

Treatments

	T_1	T_2	T_3	T_4
B_1	12	14	20	22
B_2	17	27	19	15
B_3	15	14	17	12
B_4	18	16	22	12
B_5	19	15	20	14

Blocks

3. A set of data involving four "four tropical feed stuffs A, B, C, D" tried on 20 chicks is given below. All the twenty chicks are treated alike in all respects except the feeding treatments and each feeding treatment is given to 5 chicks. Analyze the data. Weight gain of baby chicks fed on different feeding materials composed of tropical feed stuffs. (A/M 2010)

						Total
A	55	49	42	21	52	219
B	61	112	30	89	63	355
C	42	97	81	95	92	407
D	169	137	169	85	154	714
Grand Total					G = 1695

4. Four verities A, B, C, D of a fertilizer are tested in a RBD with 4 replications. The plot yields in pounds are as follows:

A12 D20 C16 B10
D18 A14 B11 C14
B12 C15 D19 A13
C16 B11 A15 D20

Analyse the experimental yield. (M/J 2012),(M/J 2014)

5. The result of an RBD experiment on 3 blocks with 4 treatments A, B, C, D are tabulated here. Carry out an analysis of variance. (M/J 2016)

	Treatment effects			
I	A36	D35	C21	B36
II	D32	B29	A28	C31
III	B28	C29	D29	A26

6. Carry out ANOVA (Analysis of variance) for the following.   (N/D 2010)

		A	B	C	D
	1	44	38	47	36
	2	46	40	52	43
Workers	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

7. The following data represent the number of units of production per day turned out by 5 different workers using 4 different types of machines.   (A/M 2011),(M/J 2013)

Machine Type

		A	B	C	D
	1	44	38	47	36
	2	46	40	52	43
Workers	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

(i) Test whether the mean production is the same for the different machine types.

(ii) Test whether the 5 men differ with mean productivity.

8. The sales of 4 salesmen in 3 seasons are tabulated here. Carry out an analysis of variance.   (N/D 2012)

Salesmen

Seasons	A	B	C	D
Summer	36	36	21	35
Winter	28	29	31	32
Monsoon	26	28	29	29

- ## **Latin Square (Three – Way Classification)**

1. A variable trial was conducted on wheat with 4 varieties in a Latin Square design. The plan of the experiment and per plot yield are given below:

D25	B23	A20	D20
A19	D19	C21	B18
B19	A14	D17	C20
D17	C20	B21	A15

Analyse the data.   (M/J 2012)

P.KUPPAN M.Sc., B.Ed., M.Phil

2. A farmer wishes to test the effect of 4 fertilizers A, B, C, D on the yield of wheat. The fertilizers are used in a LSD and the result are tabulated here. Perform an analysis of variance. (N/D 2012)

A18	C21	D25	B11
D22	B12	A15	C19
B15	A20	C23	D24
C22	D21	B10	A17

3. Analyse the following of Latin square experiment. (M/J 2013)

A12	D20	C16	B10
D18	A14	B11	C14
B12	C15	D19	A13
C16	B11	A15	D20

4. The following is a Latin square of a design when 4 varieties of seed are being tested. Set up the analysis of variance table and state your conclusion. You can carry out the suitable charge of origin and scale. (N/D 2013)

A 110	B 100	C 130	D 120
C 120	D 130	A 110	B 110
D 120	C 100	B 110	A 120
B 100	A 140	D 100	C 120

5. Analyse the variance in the Latin square of yields (in kgs) of paddy where P,Q, R, S denote the different methods of cultivation: (M/J 2014)

S122	P121	R123	Q122
Q124	R123	P122	S125
P120	Q119	S120	R121
R122	S123	Q121	P122

Examine whether different method of cultivation have significantly different yields.

P.KUPPAN M.Sc., B.Ed., M.Phil

6. In a Latin square experiment given below are the yields in quintals per acre on the paddy crop carried out for testing the effect of five fertilizers A, B, C, D, E. Analyze the data for variations. (A/M 2011)

B 25	A 18	E 27	D 30	C 27
A 19	D 31	C 29	E 26	B 23
C 28	B 22	D 33	A 18	E 27
E 28	C 26	A 20	B 25	D 33
D 32	E 25	B 23	C 28	A 20

7. The following is a Latin square of a design when 4 varieties of seeds are being tested. Set up the analysis of variance table and state your conclusion. You may carry out suitable change of origin and scale. (M/ J 2013)

A 105	B 95	C 125	D 115
C 115	D 125	A 105	B 105
D 115	C 95	B 105	A 115
B 95	A 135	D 95	C 115

8. A company wants to produce cars for its own use. It has to select the make of the car out of the four makes A, B, C and D available in the market. For this he tries four cars of each make by assigning the cars to four drivers to run on four different routes. The efficiency of cars is measured in terms of time in hours. The layout and time consumed is as given below.

		Drivers		
Routes	1	2	3	4
1	18 (C)	12 (D)	16 (A)	20 (B)
2	26 (D)	34 (A)	25 (B)	31 (C)
3	15 (B)	22 (C)	10 (D)	28 (A)
4	30 (A)	20 (B)	15 (C)	9 (D)

Analyse the experimental data and draw conclusions. ($F_{0.05}(3,5) = 5.41$) (N/D 2014)

- ## Theory Questions

  1. Compare and contrast the Latin square Design with the Randomised Block Design.
     (M/J 2013)
  2. What are the basic assumptions involved in ANOVA? (A/M 2011)

Unit – III (Solution of Equations and Eigenvalue Problems)

- ## Newton – Raphson method

1. Using Newton-Raphson method, solve $x \log_{10} x = 12.34$ taking the initial value x_0 as 10.
   (M/J 2012)

2. Solve the equation $x \log_{10} x = 1.2$ using Newton-Raphson method. (M/J 2014)

3. Find the real positive root for the equation $3x - \cos x = 1$ by Newton-Raphson method correct to 6 decimal places. (A/M 2011),(N/D 2013),(N/D 2017)

- ## Gauss – Jacobi method & Gauss – Seidel method

1. Find the solution, to three decimals, of the system using Gauss-Seidal method
   $8x + 11y - 4z = 95; \ 7x + 52y + 13z = 104; \ 3x + 8y + 29z = 71$. (N/D 2014)
2. Solve the following set of equations using Gauss-Seidal iterative procedure
   $-10x + 2y + 2z = 4; \ x - 10y + 2z = 18; \ x + y - 10z = 45$. (M/J 2014)
3. Solve the following system of equations using Gauss-Seidal iterative method
   $27x + 6y - z = 85, \ 6x + 15y + 2z = 72, \ x + y + 54z = 110$. (M/J 2012)
4. Solve the following equations by Gauss-Seidal method
   $x + y + 54z = 110, \ 27x + 6y - z = 85, \ 6x + 15y + 2z = 72$.

   (A/M 2011),(N/D 2012),(N/D 2017)

5. Solve $5x - y + z = 10; \ 2x + 4y = 12$ and $x + y + 5z = -1$ using Gauss Seidel method.
   (A/M 2010)

6. Solve by Gauss-Seidel method $6x + 3y + 12z = 35; \ 8x - 3y + 2z = 20;$
   $4x + 11y - z = 33$. (N/D 2010)

7. Solve by Gauss-Seidal method $28x + 4y - z = 32; \ x + 3y + 10z = 24;$
   $2x + 17y + 4z = 35$. (M/J 2013)

8. Solve by Gauss-Seidal method $x + y + 9z = 15; \ x + 17y - 2z = 48;$
   $30x - 2y + 3z = 75$. (M/J 2016)

- ## **Gauss Elimination method & Gauss Jordan method**

1. Solve the following equations using Gauss-elimination method
   $2x + y + 4z = 12,\ 8x - 3y + 2z = 20,\ 4x + 11y - z = 33$. (M/J 2016)

2. Solve the system of equations using Gauss-elimination method
   $5x - 2y + z = 4,\ 7x + y - 5z = 8,\ 3x + 7y + 4z = 10$. (N/D 2014)

3. Solve the following equations by Gauss elimination method
   $x + y + z = 9,\ 2x - 3y + 4z = 13,\ 3x + 4y + 5z = 40$. (N/D 2012)

4. Solve the following system of equation by Gauss elimination method:
   $2x + y + z = 10;\ 3x + 2y + 3z = 18;\ x + 4y + 9z = 16$. (A/M 2010)

5. Solve by Gauss-Elimination method $3x + 4y + 5z = 18;\ 2x - y + 8z = 13;$
   $5x - 2y + 7z = 20$. (M/J 2013)

6. Using Gauss-Jordan, solve the following system $10x + y + z = 12;\ 2x + 10y + z = 13;$
   $x + y + 5z = 7$. (N/D 2010)

7. Solve the system of equations by Gauss-Jordan method $x + y + z + w = 1;$
   $2x - y + 2z - w = -5;\ 3x + 2y + 3z + 4w = 7;\ x - 2y - 3z + 2w = 5$. (M/J 2013)

8. Solve the system of equations by Gauss-Elimination method $x_1 + x_2 + x_3 + x_4 = 2;$
   $2x_1 - x_2 + 2x_3 - x_4 = -5;\ 3x_1 + 2x_2 + 3x_3 + 4x_4 = 7;\ x_1 - 2x_2 - 3x_3 + 2x_4 = 5$.
   (N/D 2013)

- ## **Matrix inversion by Gauss Jordan method**

1. Find the inverse of the matrix $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{pmatrix}$ using Gauss-Jordan method.

   (N/D 2014)

2. Using Gauss Jordon method, find the inverse of $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$.

   (A/M 2010), (M/J 2012)

3. If $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$, find A^{-1} by Gauss-Jordan method. (N/D 2012)

4. Find the inverse of $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 3 & -1 & -4 \end{pmatrix}$ using Gauss-Jordon method. (N/D 2010)

5. By Gauss-Jordan method, find the inverse of $A = \begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$.

    (A/M 2011),(M/J 2016),(N/D 2017)

6. Find the inverse of the matrix $\begin{pmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ by Gauss-Jordan method. (N/D 2013)

7. By Gauss Jordan elimination method, find the inverse of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & -1 \\ 2 & -1 & 2 \end{pmatrix}$.

    (M/J 2014)

- **Eigen values of a matrix by Power method**

1. Find the numerically largest eigenvalue of $A = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}$ by power method.

    (M/J 2012),(M/J 2014)

2. Find the dominant eigenvalue of $\begin{pmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{pmatrix}$ by power method. (N/D 2012)

3. Find the dominant eigenvalue and its eigenvector of the matrix by power method
    $A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$. (N/D 2014)

4. Using power method, find all the eigenvalues of $A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$. (M/J 2013)

5. Determine the largest eigenvalue and the corresponding eigenvector of the matrix

$A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ with the initial vector $X^{(0)} = [1,1,1]^T$.   (A/M 2010)

6. Find all the eigenvalue of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ using power method. Using

$x_1 = (1,0,0)^T$ as initial vector.   (N/D 2010)

7. Find the numerically largest eigenvalue of $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ and the corresponding

eigenvector.   (A/M 2013),(N/D 2017)

8. Using power method find the dominant eigenvalue of the matrix $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$.

(M/J 2016)

9. Find the largest eigenvalue of the matrix $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 0 \end{pmatrix}$ by power method. Also find its

corresponding eigenvector.   (N/D 2013)

Unit – IV (Interpolation, Numerical Differential & Integration)

• Lagrange's & Newton's divided difference interpolation

1. Using Lagrange's interpolation formula, find the polynomial f(x) from the following data:

(N/D 2017)

x:	0	1	4	5
f(x):	4	3	24	39

2. Using Lagrange's interpolation formula, find $y(10)$ from the following table. (A/M 2011)

x :	5	6	9	11
y :	12	13	14	16

3.   Using Lagrange's interpolation, find the value of $f(3)$, from the following table:

(M/J 2012)

$$x: \quad 0 \quad 1 \quad 2 \quad 5$$
$$f(x): \quad 2 \quad 3 \quad 12 \quad 147$$

4.   Using Lagrange's method, find the value of $f(3)$ from the following table: (N/D 2013)

$$x: \quad 0 \quad 1 \quad 2 \quad 3$$
$$f(x): \quad 2 \quad 3 \quad 12 \quad 147$$

5.   Use Lagrange's formula to fit a polynomial to the following data hence find $y(x=1)$.

(N/D 2010)

$$x: \quad -1 \quad 0 \quad 2 \quad 3$$
$$y: \quad -8 \quad 3 \quad 1 \quad 12$$

6.   Find polynomial $f(x)$ by using Lagrange's formula and hence find $f(4)$ for

(M/J 2014)

$x:$	1	3	5	7
$f(x):$	24	120	336	720

7.   Given the table of values

$x:$	50	52	54	56
$\sqrt[3]{x}:$	3.684	3.732	3.779	3.825

Use Lagrange's formula to find $\sqrt[3]{53}$.   (N/D 2014)

8.   From the following values, find f(x) and hence find f(6) by Newton's divided difference
      formula.   (N/D 2017)

x:	1	2	7	8
f(x):	1	5	5	4

9.   Use Newton divided difference formula to calculate $f(3), f'(3)$ and $f''(3)$ from the
      following table: (A/M 2010)

$x:$	0	1	2	4	5	6
$f(x):$	1	14	15	5	6	19

10.   Using Newton's divided difference formula, find the values of $f(2), f(8)$ and $f(15)$
      given the following table.   (A/M 2011),(M/J 2013),(N/D 2013)

$$x: \quad 4 \quad 5 \quad 7 \quad 10 \quad 11 \quad 13$$
$$f(x): \quad 48 \quad 100 \quad 294 \quad 900 \quad 1210 \quad 2028$$

P.KUPPAN M.Sc., B.Ed., M.Phil

11. Given the set of tabulated points $(1,-3)$, $(3,9)$, $(4,30)$ and $(6,132)$ obtain the value of y when $x = 2$ using Newton's divided difference formula.   (N/D 2014)

12. Find the cubic polynomial $y(x)$ for   (N/D 2012)

$$x: \quad -1 \quad 0 \quad 2 \quad 3$$
$$y(x): \quad -8 \quad 3 \quad 1 \quad 12$$

• Newton's forward and backward difference interpolation

1. Interpolate $y(12)$, if   (N/J 2016)

x	10	15	20	25	30	35
$y(x)$	35	33	29	27	22	14

2. Using Newton's forward interpolation formula, find the polynomial $f(x)$ satisfying the following data. Hence evaluate $f(x)$ at $x = 5$.   (M/J 2012)

$$x: \quad 4 \quad 6 \quad 8 \quad 10$$
$$f(x): \quad 1 \quad 3 \quad 8 \quad 16$$

3. Construct Newton's forward interpolation polynomial for the following data:

$x:$	1	2	3	4	5
$f(x):$	1	-1	1	-1	1

and hence find $f(3.5)$, $f'(3.5)$.   (M/J 2014)

4. Find $y(1976)$ from the following   (N/D 2010)

$x:$	1941	1951	1961	1971	1981	1991
$y:$	20	24	29	36	46	51

5. From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46 and 63.   (A/M 2011)

Age x :   45   50   55   60   65

Premium y : 114.84   96.16   83.32   74.48   68.48

6. Find $y(22)$, given that   (N/D 2012)

$$x: \quad 20 \quad 25 \quad 30 \quad 35 \quad 40 \quad 45$$
$$y(x): \quad 354 \quad 332 \quad 291 \quad 260 \quad 231 \quad 204$$

P.KUPPAN M.Sc., B.Ed., M.Phil

- ## **Approximation of derivatives using interpolation**

1. Compute $f'(0)$ and $f''(4)$ from the following data: (N/D 2012)

$$x: \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$
$$f(x): \quad 1 \quad 2.718 \quad 7.381 \quad 20.086 \quad 54.598$$

2. From the following table of values of x and y, obtain $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ for $x = 1.2$.

    (A/M 2010)

$x:$	1.0	1.2	1.4	1.6	1.8	2.0	2.2
$y:$	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

3. Find the first and second derivatives of $f(x)$ at $x = 1.5$ if (N/D 2014)

$$x: \quad 1.5 \quad 2.0 \quad 2.5 \quad 3.0 \quad 3.5 \quad 4.0$$
$$f(x): \quad 3.375 \quad 7.000 \quad 13.625 \quad 24.000 \quad 38.875 \quad 59.000$$

4. The population of a certain town is given below. Find the rate of growth of the population in 1931, 1941, 1961 and 1971. (M/J 2013)

Year $x:$ 1931   1941   1951   1961   1971

Population in thousands $y:$ 40.62   60.80   79.95   103.56   132.65

5. Find the value of $\cos(1.74)$, using suitable formula from the given data. (N/D 2017)

$x:$	1.7	1.74	1.78	1.82	1.86
$\sin x:$	0.9916	0.9857	0.9781	0.9691	0.9584

6. Find $y'(1)$, if (M/J 2016)

$$x: \quad -1 \quad 0 \quad 2 \quad 3$$
$$y(x): \quad -8 \quad 3 \quad 1 \quad 12$$

7. Find $y'(1)$, if (N/D 2012)

$$x: \quad 0 \quad 2 \quad 3 \quad 4 \quad 7 \quad 9$$
$$y(x): \quad 4 \quad 26 \quad 58 \quad 112 \quad 466 \quad 922$$

- **Numerical integration using Trapezoidal & Simpson's Rule**

1. Evaluate $\int_0^1 \dfrac{dx}{1+x}$ by using Simpson's one-third rule and hence deduce the value of

    $\log_e 2$.   (M/J 2014)

2. Evaluate $\int_0^\pi \sin x \ dx$, by trapezoidal and Simpson's $\left(\dfrac{1}{3}\right)$ rules by dividing the range into

    10 equal parts. Verify your answer with integration.   (N/D 2012),(M/J 2013)

3. Taking $h = \dfrac{\pi}{10}$, evaluate $\int_0^\pi \sin x \ dx$ by Simpson's 1/3 rule. Verify the answer with

    integration.   (N/D 2010)

4. Evaluate $\int_0^2 \dfrac{dx}{x^2+x+1}$ to three decimals, dividing the range of integration into 8 equal

    parts using Simpson's rule.   (M/J 2012)

5. Evaluate $\int_0^1 \dfrac{dx}{1+x^2}$ by Simpson's (1/3) rule, dividing the range into four equal parts.

                                                                                                         (M/J 2016)

6. Evaluate $\int_0^6 \dfrac{1}{1+x^2} \ dx$ using Trapezoidal rule. Verify the answer with direct integration.

                                                                                                         (N/D 2010)

7. The table below gives the velocity V of a moving particle at time t seconds. Find the distance covered by the particle in 12 seconds and also the acceleration at $t = 2$ seconds, using Simpson's rule.   (A/M 2011)

t :	0	2	4	6	8	10	12
V :	4	6	16	34	60	94	136

8. The velocity v of a particle at a distance s form a point on its path is given as follows:

s in meter:	0	10	20	30	40	50	60
v m/sec:	47	58	64	65	61	52	38

    Estimate the time taken to travel 60 meters by using Trapezoidal rule and Simpson's rule.   (M/J 2014)

9. The velocities of a car (running on a straight road) at intervals of 2 minutes are given below.   (N/D 2014)

Time in minutes:	0	2	4	6	8	10	12
Velocities in km/hr:	0	22	30	27	18	7	0

Apply Simpson's rule to find the distance covered by the car.

10. A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is in the table below. Using trapezoidal rule and Simpson's 1/3 rule, find the velocity of the rocket at $t = 80$ sec. (A/M 2010)

$t(\text{sec}):$	0	10	20	30	40	50	60	70	80
$t(cm / \text{sec}):$	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	40.67

- ## Double integrals by Trapezoidal and Simpsons's rules

1. Using Trapezoidal rule, evaluate $\int\limits_{1}^{2}\int\limits_{1}^{2}\dfrac{dxdy}{x^2+y^2}$ numerically with $h = 0.2$ along x - direction and $k = 0.25$ along y -direction. (M/J 2012)

2. Evaluate $\int\limits_{0}^{1}\int\limits_{0}^{1}\dfrac{1}{1+x+y}dxdy$ by trapezoidal rule. (N/D 2014)

3. Evaluate $\int\limits_{1}^{1.2}\int\limits_{1}^{1.4}\dfrac{dxdy}{x+y}$ by trapezoidal formula by taking $h = k = 0.1$.

   (A/M 2010),(N/D 2017)

4. Using Trapezoidal rule, evaluate $\int\limits_{1}^{2}\int\limits_{1}^{2}\dfrac{dxdy}{x+y}$ with $h = k = 0.5$. (M/J 2016)

5. Evaluate $\int\limits_{0}^{2}\int\limits_{0}^{1}4xy\ dxdy$ using Simpson's rule by taking $h = \dfrac{1}{4}$ and $k = \dfrac{1}{2}$.(N/D 2012)

6. Evaluate $\int\limits_{0}^{1/2}\int\limits_{0}^{1/2}\dfrac{\sin(xy)}{1+xy}dxdy$ using Simpson's rule with $h = k = \dfrac{1}{4}$.

   (M/J 2012),(M/J 2014)

7. Evaluate $\int\limits_{1}^{1.2}\int\limits_{2}^{2.4}\dfrac{1}{xy}dxdy$ using Simpon's one-third rule. (M/J 2013)

P.KUPPAN M.Sc., B.Ed., M.Phil

Unit – V (Numerical Solution of Ordinary Differential Eqns.)

• Taylor's series, Euler's and Modified Euler's method

1. Using Taylor series method, find the value of y at $x = 0.1$, if y satisfies the equation $\frac{dy}{dx} = x^2 - y$ given that $y = 1$ when $x = 0$, correct to 3 decimal places. (N/D 2017)

2. Apply Taylor series method to find and approximate value of y when $x = 0.1, 0.2$ given that $\frac{dy}{dx} = x + y, \ y(0) = 1$. (M/J 2014)

3. Given $y' = x^2 - y, \ y(0) = 1, \ y(0.1) = 0.9052, \ y(0.2) = 0.8213$, find $y(0.3)$ using Taylor's series method. (N/D 2013)

4. Using Taylor method, compute $y(0.2)$ and $y(0.4)$ correct to 4 decimal places given $\frac{dy}{dx} = 1 - 2xy$ and $y(0) = 0$, by taking $h = 0.2$. (A/M 2011)

5. By Taylor series method find $y(0.1), y(0.2)$ and $y(0.3)$ if $\frac{dy}{dx} = x - y^2, \ y(0) = 1$. (N/D 2012)

6. Given $\frac{dy}{dx} = 1 + y^2$, where $x = 0$, find $y(0.2), y(0.4)$ and $y(0.6)$, using Taylor series method. (A/M 2010)

7. Solve by Taylor's method to find an approximate value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x, \ y(0) = 0$. Compare the numerical solution with the exact solution. Use first three non-zero terms in the series. (N/D 2014)

8. Use Euler's method, with $h = 0.1$ to find the solution of $y' = x^2 + y^2$ with $y(0) = 0$ in $0 \le x \le 5$. (N/D 2010)

9. Solve by Euler's method, the equation $\frac{dy}{dx} = x + y, \ y(0) = 0$, chose $h = 0.2$ and compute $y(0.4)$ and $y(0.6)$. (N/D 2013)

P.KUPPAN M.Sc., B.Ed., M.Phil

10. Consider the initial value problem $\dfrac{dy}{dx} = y - x^2 + 1, \ y(0) = 0.5$. Compute $y(0.2)$ by Euler's method and modified Euler method. (M/J 2012),(N/D 2014)

11. Using modified Euler method, find $y(0.2), y(0.1)$ given $\dfrac{dy}{dx} = x^2 + y^2, \ y(0) = 1$.

    (A/M 2011)

12. By Modified Euler's method, find $y(0.1), y(0.2)$ and $y(0.3)$ if $\dfrac{dy}{dx} = x + y, \ y(0) = 1$.

    (N/D 2012)

13. Evaluate $y(1.2)$ and $y(1.4)$ correct to three decimal places by the modified Euler method, given that $\dfrac{dy}{dx} = \left(y - x^2\right)^3; \ y(1) = 0$ taking $h = 0.2$. (M/J 2014)

• Runge – Kutta method for 1st order equations

1. Find $y(0.8)$ given that $y' = y - x^2, \ y(0.6) = 1.7379$ by using R-K method of order 4, taking $h = 0.1$. (M/J 2012)

2. Using 4th order Runge-Kutta method, solve $\dfrac{dy}{dx} = \dfrac{y^2 - x^2}{y^2 + x^2}, \ y(0) = 1$ for $x = 0.2$ and $x = 0.4$ with $h = 0.2$. (A/M 2010),(A/M 2011),(M/J 2013)

3. Use R.K Method fourth order to find the $y(0.2)$ if $\dfrac{dy}{dx} = x + y^2, \ y(0) = 1, \ h = 0.1$.

    (N/D 2010),(N/D 2017)

4. Solve $\dfrac{dy}{dx} = xy + y^2, y(0) = 1$, for $y(0.1), \ y(0.2)$, using fourth order Runge-Kutta method. (N/D 2014)

• Milne's predictor – corrector methods for 1st order eqn.

1. Given $\dfrac{dy}{dx} = x^3 + y, \ y(0) = 2, \ y(0.2) = 2.073, \ y(0.4) = 2.452, \ y(0.6) = 3.023$, compute $y(0.8)$ by Milne's method. (N/D 2017)

2. If $\dfrac{dy}{dx} = \dfrac{y^2 - x^2}{y^2 + x^2}, \ y(0) = 1$, find $y(0.2), y(0.4)$ and $y(0.6)$ by Runge-Kutta method of fourth order and hence find $y(0.8)$ by Milne's method. (N/D 2012), (M/J 2016)

3. Compute $y(0.4)$ and $y(0.5)$, given that $y' = y - \dfrac{2x}{y}, \ y(0) = 1, \ y(0.1) = 1.0954$, $y(0.2) = 1.1832, \ y(0.3) = 1.2649$ using Milne's predictor-corrector method.

    (N/D 2014)

P.KUPPAN M.Sc., B.Ed., M.Phil

4. If $\dfrac{dy}{dx} = x^2 + y^2$, $y(0) = 1$ find $y(0.1)$, $y(0.2)$ and $y(0.3)$ by Taylor series method. Hence find $y(0.4)$ by Milne's Predictor-Corrector method. (M/J 2016)

5. Using Milne's predictor and corrector method find $y(4.4)$ given $5xy' + y^2 - 2 = 0$ given $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$, $y(4.3) = 1.0143$.
(M/J 2012),(M/J 2014)

6. Using Milne's method, obtain the solution of $\dfrac{dy}{dx} = x - y^2$ at $x = 0.8$ given $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$. (N/D 2010)

7. Using Milne's predictor-corrector method, find $y(0.4)$, given that $y' = \dfrac{\left(1 + x^2\right)y^2}{2}$, $y(0) = 1$, $y(0.1) = 1.06$, $y(0.2) = 1.12$, $y(0.3) = 1.21$. (N/D 2013)

• Finite difference methods for solving 2nd order equation

1. Solve the equation $\dfrac{d^2 y}{dx^2} = x + y$ with boundary conditions $y(0) = 1 = y(1)$ by finite difference method, by taking 4 subintervals. (N/D 2017)

2. Solve the BVP $\dfrac{d^2 y}{dx^2} - y = 0$, with $y(0) = 0$, $y(1) = 1$, using finite difference method with $h = 0.2$. (M/J 2012)

3. Solve the BVP $y'' + y = 0$, $y(0) = 1$, $y(1) = 0$ using finite difference method, taking $h = 0.25$. (M/J 2014)

4. Solve the differential equation $\dfrac{d^2 y}{dt^2} + y = \sin 2t$; $y(0) = 0$, $y'(0) = 0$ by using Laplace transform method. (N/D 2009)

5. Using finite difference solve the boundary value problem $y'' + 3y' - 2y = 2x + 3$, $y(0) = 2$, $y(1) = 1$ with $h = 0.2$. (A/ M 2010)

6. Solve $y_{x+2} - 7y_{x+1} - 8y_x = x(x-1)2^x$. (M/J 2013)

----*All the Best*----

P.KUPPAN M.Sc., B.Ed., M.Phil