Reg. No. :

Question Paper Code : 80213

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Third Semester

Medical Electronics

MA 8352 — LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Biomedical Engineering/Computer and Communication Engineering/Electronics and Communication Engineering/Electronics and Telecommunication Engineering)

(Regulation 2017)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Determine whether the vectors $v_1 = (1, 2, 1), v_2 = (2, 1, 0)$ and $v_3 = (1, -1, 2)$ form a linearly independent or linearly dependent in $V_3(R)$.

2. If $V = A \oplus B$, then show that dim V = dim A+ dim B,

3. Define Kernal of T.

4. Obtain the matrix representing the linear transformation $T : V_3(R) \to V_3(R)$ given by $T(a, b, c) = (3a, a-b, 2a+b+c)$ with respect to the standard basis $\{e_1, e_2, e_3\}$.

5. Let V be the vector space of polynomials with linear product given by $\langle f, g \rangle = \int_0^1 f(t) g(t)\, dt$ where $f(t) = t+2$ and $g(t) = t^2 - 2t - 3$. Find $\langle f, g \rangle$.

6. Define Adjoint matrix.

7. Find the differential equation of all spheres whose centres lie on the Z- axis.

8. Solve $p^2 + q^2 = x + y$.

9. Write the formula for Half range Fourier sine series.

10. A slightly stretched string of length l has its ends fastened at $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = y_0 \sin \frac{3\pi x}{l}$. If it is released from rest from this position, write the boundary conditions.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Let R^+ be the set of all positive real numbers. Define addition and scalar multiplication as follows $u + v = uv$ for all $u, v \in R^+$; $\alpha u = u^\alpha$ for all $u \in R^+$ and $\alpha \in R^+$. Determine whether or not R^+ is a real vector space.

(ii) Prove that $S = \{v_1, v_2, \cdots, v_n\}$ is a linearly dependent set of vectors in V iff there exists a vector $v_k \in S$ such that v_k is a linear combination of the preceding vectors $v_1, v_2, \cdots v_n$. (6 + 10)

Or

(b) (i) If $\alpha_1, \alpha_2, \alpha_3$ are fixed elements of a field F, then show that the set W of all ordered triads $\{x_1, x_2, x_3\}$ of elements of F, such that $\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 = 0$, is a sub space of $V_3(F)$.

(ii) Prove that every linearly independent subset of a finitely generated vector space V (F) is either a basis of V or can be extended to form a basis of V. (8 + 8)

12. (a) (i) Find the linear transformation $T : V_3(R) \to V_3(R)$ determined by the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ write the standard basis $\{e_1, e_2, e_3\}$.

(ii) Let V be a vector space over a field F. Let A and B be subspaces of V, then show that $\frac{A+B}{A} \cong \frac{B}{A \cap B}$. (6 + 10)

Or

(b) (i) Let L be a linear transformation from R^3 to R^3 whose matrix representation A with respect to the standard basis is given below. Find the Eigenvalues of L and a basis of Eigenvectors

$$A = \begin{pmatrix} 1 & 3 & -3 \\ 3 & 1 & -3 \\ -3 & -3 & 1 \end{pmatrix}.$$

(ii) If A is an m × n matrix, then prove that $N(A)$ is a sub space of R^n. (8+ 8)

13. (a) (i) Let V be the set of all continuous real valued functions defined on the closed interval [0, 1], then prove that V is a real inner product space with inner product, defined by

$$< f, g >= \int_0^1 f(t)\, g(t)\, dt.$$

(ii) Find the orthogonal basis containing the vector (1, 3, 4) for $V_3(R)$ with the standard inner product. (8 + 8)

Or

(b) State and prove Gram - Schmidt orthogonalisation process. (16)

14. (a) (i) Form the partial differential equation by eliminating the arbitrary functions f and ϕ from $Z = x\, f(y/x) + y\, \phi(x)$,

(ii) Solve $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x\, \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x+y}$. (8 + 8 = 16)

Or

(b) (i) Solve $(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$.

(ii) Solve $p^2 + q^2 = x^2 + y^2$. (8 + 8)

15. (a) (i) Express $f(x) = (\pi - x)^2$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$.

(ii) Show that in $0 \le x \le \pi$, $x(\pi - x) = \frac{\pi^2}{6} - \left(\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \cdots\right)$. (8+ 8)

Or

(b) The points of trisection of a string are pulled aside through the same distance on opposite sides of the position of equilibrium and the string is released from rest. Derive an expression for the displacement of the string at subsequent time and show that the mid-point of the string always remains at rest. (16)